Scaling Two- & Three-Dimensional Figures

Generally, when multiplying each length of a two- or three-dimensional figure by a factor (for instance, 3), square the factor to find the new area. For example, if you start with a unit square (1 unit by 1 unit) and multiply the length of each side by a factor (for instance, 3), the new area will be 3^2, or 9 square units. In a unit cube, or three-dimensional 1-unit-by-1-unit square, the original surface area is equal to 6 square units (1 square unit per side times 6 sides). The new surface area is 6 x 9 or 54 square units.

In a rectangular prism like the one I had on the show (the tissue box), the original surface area was 2 x 1 x 1 (the opposite ends) + 4 x 3 x 1 (the four congruent sides between the ends) = 2 + 12 = 14 square units. The volume of the rectangular prism was 1 x 3 x 1 = 3 cubic units.

When I multiplied the boxes (the 6 boxes wrapped together), the two new ends were 2 x 3 = 6 square units each, the new front and back were 3 x 3 = 9 square units each and the new top and bottom were 2 x 3 = 6 square units each. That means the new surface area of the 6 boxes was 2 x 6 (ends) + 2 x 9 (front and back) + 2 x 6 (top and bottom) = 12 + 18 + 12 = 42 square units. The volume of the 6 wrapped boxes was 2 x 3 x 3 = 18 cubic units. Even though the area was 42/14 = 3 times larger, the volume was 18/3 = 6 times larger. The change in the volume is understandable. Multiply the number of boxes by six, and the volume is 6 times larger. The change in the surface area is not so predictable because of the surfaces of the individual boxes that are inside the larger figure and not part of the surface area of the larger figure.

Changing surface area and volume are difficult concepts, and not necessarily intuitive. Children need practice using them.