

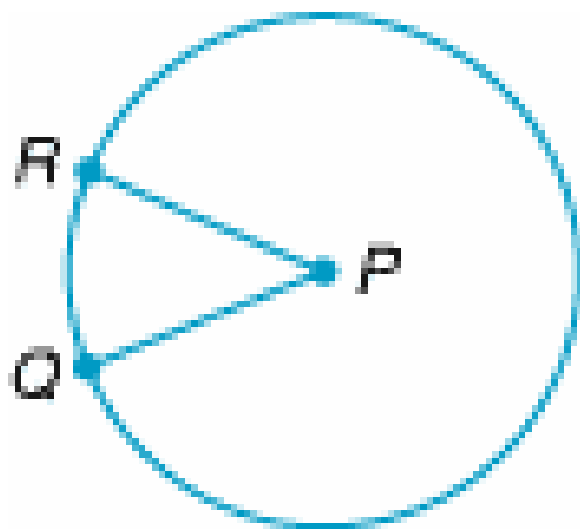


Glencoe Geometry Chapter 9.3 and 9.4

## Circles: Arcs, Chords, Inscribed Angles, and Area

Last week we learned how to find the degree *measure* of an arc of a circle—it was the same as the degree measure of the intercepted central angle.

For instance, in this circle, in  $\odot P$ , if  $\overline{PR} = 9\text{in}$ ,  $m\angle QPR = 40$ , then  $m\widehat{QR} = 40$  too.

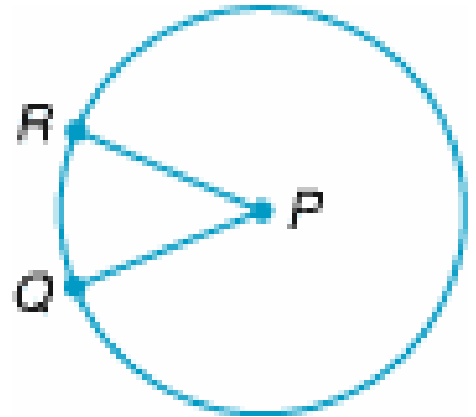


The *arc length*,  $s$ , measured in a linear unit, is a part of the circumference that is proportional to the measure of the central angle compared to the entire circle.

$$\frac{\text{part of circumference}}{\text{whole circumference}} = \frac{\text{degree part of circle}}{\text{degree of whole circle}}$$

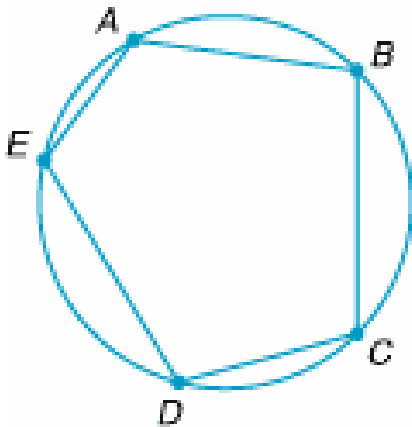
$$\frac{\text{arc length}}{2\pi r} = \frac{\text{central angle}}{360^\circ}$$

$$\begin{aligned} \frac{s}{2\pi(9)} &= \frac{40^\circ}{360^\circ} \\ \frac{s}{18\pi} &= \frac{1}{9} \\ s &= \frac{18\pi}{9} = 2\pi \approx 6.283 \text{ in} \end{aligned}$$



Given a circle, we can inscribe either a polygon or an angle inside of it.

A polygon is said to be inscribed if each of its vertices lies on the circle. **For instance, polygon  $ABCDE$  is an inscribed pentagon.** The line segments are chords.



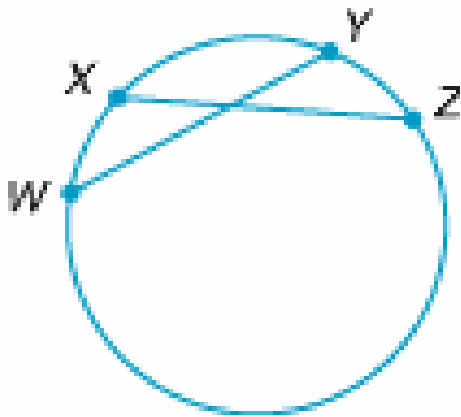
Notice that an arc can be defined, not only by a central angle, but also by a chord.

Here's an important theorem relating two minor arcs to their respective chords.

**Theorem:** In a circle (or congruent circles), two or more arcs are congruent if and only if their corresponding chords are congruent.

**Example:**

In the circle,  $m\widehat{WX} = 30$ ,  $m\widehat{XY} = 50$ ,  $m\widehat{YZ} = 30$ , and  $WY = 14$ . Find  $XZ$ .



$$m\widehat{WY} = m\widehat{WX} + m\widehat{XY}$$

$$m\widehat{WY} = 30 + 50 = 80$$

Now we show that  $\widehat{WY} \cong \widehat{XZ}$ :

$$m\widehat{XZ} = m\widehat{XY} + m\widehat{YZ}$$

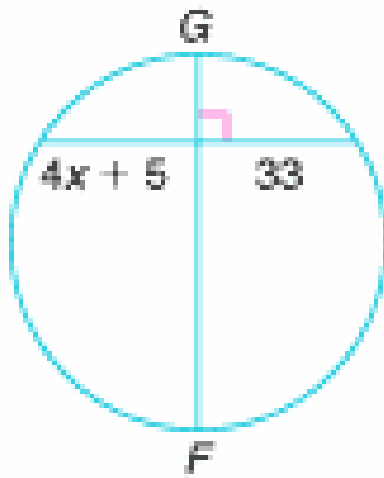
$$m\widehat{XZ} = 50 + 30 = 80 = m\widehat{WY}$$

By the theorem, since the arcs are congruent, the corresponding chords are too, so  $XZ = WY = 14$

Here's another important theorem relating a circle's diameter to a special chord.

**Theorem:** If a diameter is perpendicular to a chord, then it bisects the chord and its arc.

**Example:**



If  $\overline{GF}$  is a diameter of the circle, find the value of  $x$ .

Notice that the diameter forms a right angle with the chord, so it forms a perpendicular bisector to the chord.

From that, we can write the following algebraic equation:

$$4x + 5 = 33$$

$$4x = 28$$

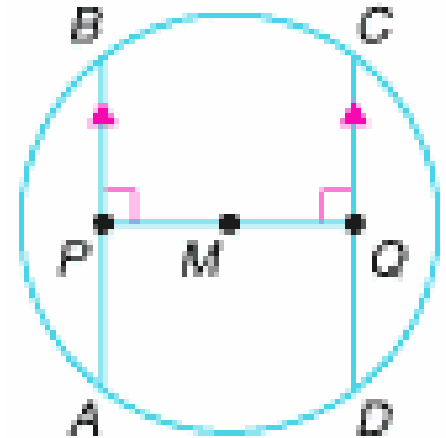
$$x = 7$$

**Theorem:** Two chords are congruent if and only if they are equidistant from the center.

**Example:**

In  $\odot M$ ,  $\overline{AB} \parallel \overline{CD}$ , and  $\overline{AB} \cong \overline{CD}$ .

If  $MP = 3$ , and  $CD = 8$ , what is the radius of the circle?



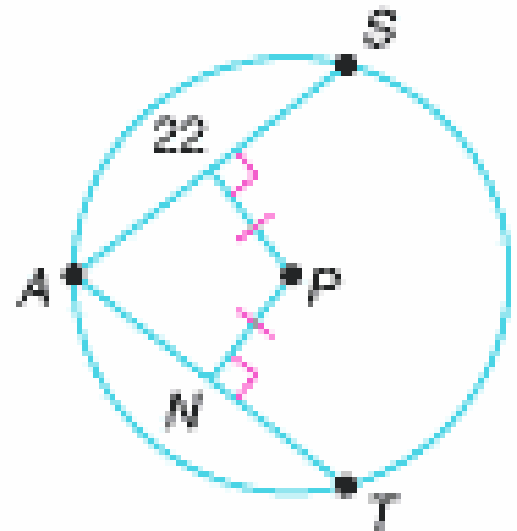
IF we extend segment  $PQ$  out on each end to the circle, we will have created a diameter, which bisects the two congruent chords. From this we know that  $CQ = DQ = 4$ . From the new theorem, we know that  $MP = MQ = 3$ . Drawing a line segment from points  $M$  to  $C$ , we have created a radius, which is also the hypotenuse of a right triangle. Using the Pythagorean Theorem and solving for the radius, we get the following:

$$r = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5$$

**Example:**

Find  $NT$

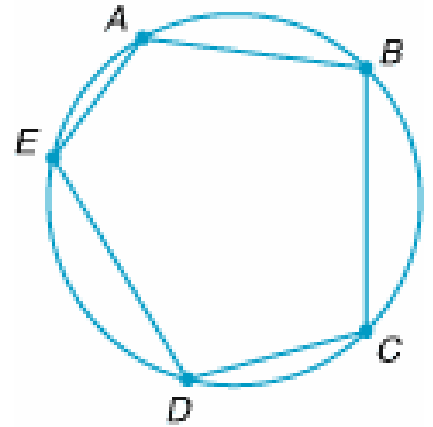
This circle has MANY congruent segments. We just need to string the correct ones together.  $AS = AT$  (same dist from center). So  $AT = 22$ . Now, segment  $NP$  is part of a diameter that is a perpendicular bisector to segment/chord  $AT$ . So,  $NA = NT = 22/2 = 11$



Angles can also be inscribed in a circle.

An inscribed angle has its vertex on the circle and its two sides are chords of a circle.

Each of the angles in this inscribed pentagon are also inscribed angles. For instance,  $\angle EAB$  or  $\angle BCD$  are inscribed angles.



Each inscribed angle has an intercepted arc. For example, the intercepted arc of  $\angle EAB$  is  $\widehat{EDB}$  or  $\widehat{ECB}$ .

Can we tell anything about the intercepted arc from its corresponding inscribed angle?? But of course . . .

If an angle is inscribed in a circle, then the measure of the angle equals one-half the measure of the intercepted arc (or the arc's measure is twice the angle's measure).

**Example:**

If  $P$  is the center of the circle, find the value of  $x$ .

$\widehat{AB}$  is subtended by two angles: central angle  $APB$  and inscribed angle  $ACB$ .

We know that  $m\widehat{AB}$  is equal to the measure of the central angle, so  $m\widehat{AB} = 4x + 4$

We also now know that  $m\widehat{AB}$  is equal to twice the measure of the inscribed angle, so  $m\widehat{AB} = 2(3x - 21)$

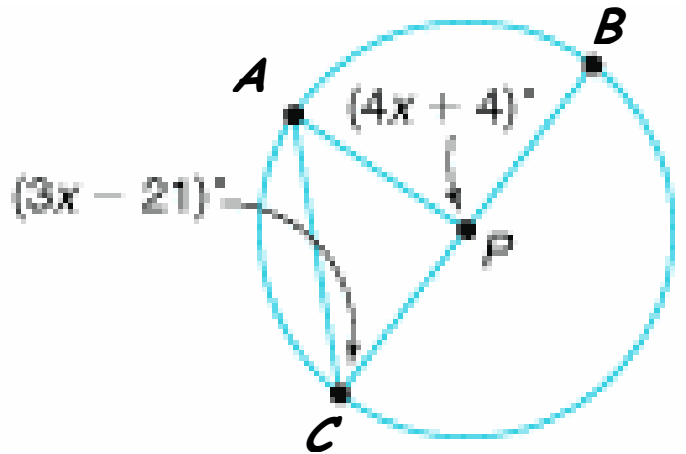
Setting these two algebraic expressions equal to each other and solving for  $x$ , we get the following:

$$4x + 4 = 2(3x - 21)$$

$$2x + 2 = 3x - 21$$

$$23 = x$$

$$x = 23$$

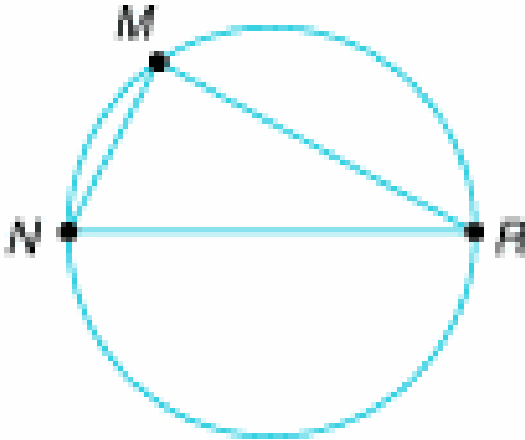


One last important theorem for today.

**Theorem:** If an inscribed angle of a circle intercepts a semicircle, then the angle is a right angle.

## Example:

If  $\overline{NR}$  is a diameter of the circle,  $MR = 12$ , and the circle has radius 10, then find  $NM$ .



Since segment  $\overline{NR}$  is a diameter, it separates the circle into two arcs that are semicircles. This means that angle  $\angle NMR$  is inscribed in a semicircle and is, therefore, a right angle. It follows that triangle  $\triangle NMR$  is a right triangle.

If the radius is 10, the diameter is 20. So  $NR = 20$ .

We are given that  $MR = 12$

We will find  $NM$  by the Pythagorean Theorem.

$$(NM)^2 + (MR)^2 = (NR)^2$$

$$(NM)^2 + 12^2 = 20^2$$

$$(NM)^2 + 144 = 400$$

$$(NM)^2 = 400 - 144 = 256$$

$$NM = \sqrt{256} = 16$$

# Say What?!?!

The Area,  $A$ , of a circle is a measure of how much space it takes up in a plane. Its formula is given by the following:

$$A = \pi r^2$$

We have used this formula several times on previous episodes, but where did it come from? Do these things fall from the sky? Do they grow on trees? Guess who discovered it . . . .

## ARCHIMEDES

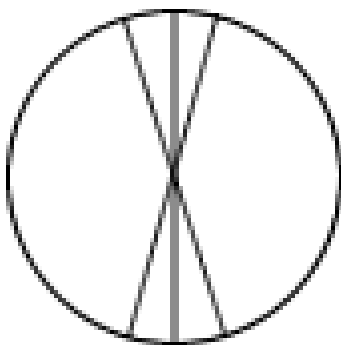
Here's the real story behind his inspiration:

One sunny afternoon, **Kyle and Wendy**, two ancient toga tailors, asked Archimedes if he was interested in walking down to a very popular pizza parlor in downtown Syracuse, Sicily for lunch. Kyle and Wendy wanted to repay Archimedes for his kindness in helping them develop a lightweight, yet durable, toga material that left the wearer cool in the summer and warm in the winter. Even though Archimedes was extremely busy, he did not want to upset his friends so he accepted their lunch invitation.

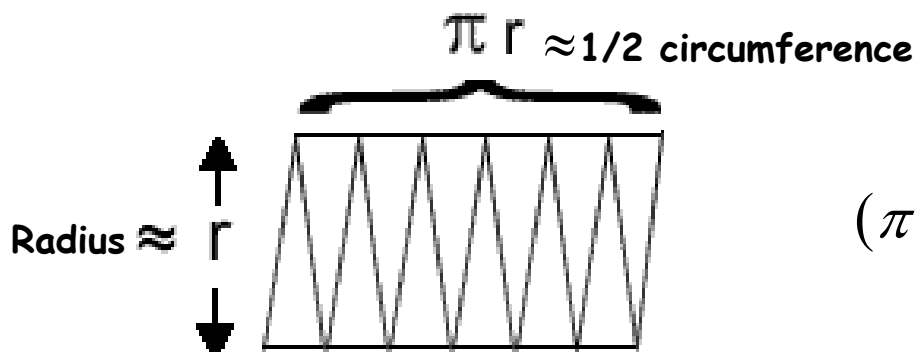


They place an order for a large, 14-inch round pizza, but just as it was served, Kyle and Wendy get summoned on a Toga service call: ***Toga emergency***. They immediately jump from their seats to answer the call for comfort, leaving Archimedes alone with the pizza.

Not interested in service calls and not really hungry, Archimedes calls to the chef, "***Hey, Rosseti, bringa me a biga, sharpa knife!***" Not understanding why Archimedes wants the knife, the chef obliges, and watches in horror while Archimedes begins to cut the round pizza into very thin slices. Each slice of pizza was uniform in size and the slices were even in number.



Now, for every slice, there is a "***Top***" and a "***Bottom***". Placing a top and bottom together, side by side, Archimedes discovered the circle becomes rectangular. The area of the near rectangular shape is approximately



$$(\pi r)(r) = \pi r^2$$

***Voilà!***

## Resources:

[www.glencoe.com](http://www.glencoe.com)

<http://www.worsleyschool.net/science/files/circle/area.html>